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Misalignment and Muon Scale Corrections Extracted from the 2011A $Z \rightarrow \mu\mu$ Sample

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Abstract

We use the 2011A Drell-Yan $\mu^+\mu^-$ data sample in the Z Mass Region ($60 < M_{\mu\mu} < 120$ GeV/c^2) to obtain corrections to the muon momentum. These corrections, extracted using a new technique, compensate for misalignments of the CMS detector. We find that the misalignments in data and Monte Carlo are different and extract corrections from both samples. The samples used for the study correspond to 2.1 fb^{-1} of integrated luminosity collected until August, 22, 2011 in pp Collisions at $\sqrt{s}$=7 TeV (referred to as the 2011A data set). The corrections to the muon momentum in both data and MC are extracted as a function of muon charge (Q) , η and ϕ.

Version 1

1 Introduction

We use the 2011A Drell-Yan $\mu^+\mu^-$ data sample in the Z Mass Region ($60 < M_{\mu\mu} < 120$ GeV/c^2) to obtain corrections to the muon momentum to compensate for misalignments of the CMS detector. The samples used for the study correspond to 2.1 fb^{-1} of integrated luminosity collected until August, 22, 2011 in pp Collisions at $\sqrt{s}$=7 TeV (referred to as the 2011A data set). We use a new technique that has not been used before.

The CMS reconstruction software in both data and MC uses incorrect alignment geometries of the tracker. The misalignments in the data and the MC are different from each other. This affects the momentum determination of muons in both data and MC. The misalignment of the tracker results in a charge (Q), η, and ϕ dependence of the determination of the muon momentum.

Although both data and MC samples were processed using the latest alignment geometry (as of December 2011) we find that the misalignment in the tracker is not fully accounted for. To correct for the remaining misalignment effect, an CMS official momentum correction (MuscleFit) was developed by the tracking group. The MuscleFit correction is parametrized using Ansatz functions. The Ansatz functions should correct for the residual charge, η, and ϕ dependence in the determination of the muon momentum. The functional forms are complicated and are different for data and MC.

The MuscleFit correction has been updated up to first 750 pb^{-1} of the 2011 data (though not approved yet) [1]. When we apply the MuscleFit correction from the first 750 pb^{-1} to the entire 2011 data set, we find that it does a poor job in correcting for misalignments (as described in an appendix to this note). There is no MuscleFit available for entire 2011 MC set. When we apply the 2010 MC MuscleFit to the 2011 MC, it also does a very poor job. We find that the MuscleFit correct for misalignments in ϕ but does not correct for misalignments in η

In this note we use a new technique to extract corrections the muon muon momentum from the average of $1/p_T$ ($< 1/p_T >$) spectra of muons from Z decays. The corrections are extracted as a function of charge, η, and ϕ. We will refer to this correction as the Rochester momentum correction. The Rochester Momentum Correction described in this note should be applied 2011A data without the application of the MuscleFit correction (since no MuscleFit exists for the 2011A data).

As we show in this communication, the Rochester Momentum Correction corrects for all of the misalignments and no MuscleFit is needed.

However, in the future, when a MuscleFit for the 2011A data becomes available, we can repeat the study and extract an updated Incremental Rochester Correction that only corrects for residual misalignment that are not fully compensated for by the yet to be developed 2011A MuscleFit. In that case, analyses which plan to use the MuscleFit can apply the Incremental Rochester Correction.

2 Data Set and Event Selection

For the extraction of corrections to the muon momentum we use 2011A data set which corresponds to 2.1 fb^{-1} of integrated luminosity. We use events that pass the HLT DoubleMu_7 trigger path. The sample is produced using the CMSSW_4_2_8 version. The Jason file is required to select the runs which satisfy the good detector condition.

The MC set which is used is the $Z \to \mu\mu$ Powheg sample of Summer 11 version which includes Pythia parton showering. The analysis selection criteria are those proposed by the Vector Boson Task Force.

These are outlined in: https://twiki.cern.ch/twiki/bin/viewauth/CMS/VbtfZMuMuBaselineSelection.

In the definition of isolation, we use the combined track and HCAL fractional isolation defined as $(TrkIso + HadIso)_{\Delta R<0.3}/P_T < 0.15$ (as used by the Dilepton group). If the EM energy is not included in the isolation requirement, then the momentum dependence of the efficiency is expected to be constant. Therefore, we do not include the EM energy in the definition of isolation.

Note that if the EM energy is included in the isolation requirement, then FSR photons cause in a momentum dependence of the efficiency, as well as a complicated correlation between the efficiency of the two muons.

Specifically, the selection criteria are:

- HLT DoubleMu_7
- Muon selection : VBTF muon selection is applied

- Pt>20 GeV and detector $|\eta| < 2.4$
- Global and Tracker Muon
- Combined relative isolation : $(TrkIso + HadIso)_{\Delta R<0.3}/P_T < 0.15$
- Global muon normalized fit $\chi^2 < 10$
- Number of Tracker hits greater than 10
- Number of pixel hits greater than or equal to 1
- Number of muon stations greater than or equal to 2
- $dxy < 0.2$
- Mass selection : $60 < Mass < 120\ GeV/c^2$

The muon reconstruction efficiency as a function of η is extracted from the data. An efficiency scale factor obtained from the data is applied to the MC to correct for the difference of the efficiency between data and MC.

3 Reference Plots Used in the Muon Momentum Study

A misalignment of the tracker generates distortions in several kinematic distributions of Drell-Yan ($Z/\gamma^* \to \mu\mu$) events in the Z boson mass region. Since the misalignment of data and MC are different, the distributions will be distorted in different ways for data and MC. Detector misalignment results in the following:

- It is responsible for charge (Q) , η, and ϕ dependence of the reconstructed Z boson mass. The expected Z boson mass is known from the generated (post FSR) spectrum in MC.
- It yields difference in the overall shape of the Z mass distributions between the data and MC (if data and MC have different misalignments). A difference in shape will also occur if the detector resolution in the MC is not modeled correctly.
- A charge dependence in the reconstructed muon momentum creates unphysical wiggles in the forward and backward charge asymmetry (A_{fb}) of Drell-Yan events as a function of dilepton mass (in the region of the Z peak). This yields one of two powerful checks on a difference in the momentum scale between positive and negative muons.
- In the low Z boson P_T region ($P_T < 10\ GeV/c$), the ϕ distribution in the Collins-Soper frame (CS) [3], ϕ_{CS}, is expected to be flat. However, resolution smearing in the muon momentum creates an excess around $\phi_{CS} = 0$ and $\pm\pi$ in the reconstructed level ϕ_{CS}. The level of the excess at $\phi_{CS} = 0$ and $\pm\pi$ is expected to be the same if the muon momentum scales and resolutions are the same between μ^+ and μ^-. Therefore, ϕ_{CS} distribution in low Z P_T region provides the second powerful check on a difference in the momentum scale between positive and negative muons. A simple way to think about is the following. ϕ_{CS} is the angle between the direction of the Z boson P_T and the direction of the positive lepton. For Z $P_T = 0$ there is no preferred x axis. However, if the calibration of the positive and negative muons are different, $P_T = 0$ events end up with a small P_T along either the positive or the negative muon direction in ϕ.

In our study we use the following kinematic distributions as reference plots to test the validity of the momentum corrections.

- The overall dimuon invariant mass spectrum ($M_{\mu^+\mu^-}$).
- A_{fb} as a function of mass.
- ϕ_{CS} in two Z P_T bins: $0 < P_T < 5\ GeV/c$, and $5 < P_T < 10\ GeV/c$.
- A comparison of the Z P_T spectrum between data and MC.
- The average Z mass as a function of ϕ of either the μ^+ or the μ^-

- The average Z mass as a function of η of either the μ^+ or the μ^-.

Figure 1 and 2 show the reference plots before any muon momentum correction to either data or MC.

The following features are observed in Fig, 1:

- The top two plots indicate that the location of the Z peak in mass is incorrect and the shape of the data in mass is different from the MC.
- The left middle plot shows unphysical wiggles in A_{fb} in both data and MC, indicating that the momentum scales for positive and negative muons are different in both data and MC
- The right middle plot shows that the MC does not have the correct P_T spectrum.
- The bottom two plots show that the MC does not have the correct P_T spectrum (level) and that the momentum scales for positive and negative muons are different (the peaks at $\phi_{CS} = 0$ and $\pm\pi$ are different).

The following features are observed in Fig. 2:

- The top two plots show that the average Z mass depends on ϕ (it should be independent of ϕ). They also show that the momentum scales for positive and negative muons are different in both data and MC, and the difference is a function of ϕ of the muon.
- The bottom two plots show that the η dependence of the muon momentum scales in data and MC are different.

4 Muon Momentum Correction- First Iteration

To correct for the effect of track misalignments, we apply a correction to the muon momentum which is a function of charge (Q), η, and ϕ of the muon.

The procedure is to require that the mean of 1/p_T ($< 1/p_T >$) of muons in data (reconstructed) and MC (reconstructed) in bins of Q, η, and ϕ should both be equal to the $< 1/p_T >$ of the MC at the generated level. Since the Z mass is known, and the P_T spectrum in MC can be tuned to agree with the data, this procedure yields an absolute calibration of the momentum scale.

In general, an overall momentum scale (e.g. error in the B field) should be the same for positive and negative muons. A misalignment would results in a difference in the mean $< 1/p_T >$ between positive and negative muon. A muon momentum correction that corrects for a misalignment is additive in 1/p_T.

As discussed later in this note, we find that the momentum correction originates from misalignment and is therefore additive in 1/p_T. After applying the additive 1/p_T momentum correction, we apply overall scale factors to the MC (which are common to positive and negative muons) to match the Z peak positions and width in data and MC..

The correction factor is defined as the difference in the mean $< 1/p_T >$ between reconstructed data (or reconstructed MC) and the mean $< 1/p_T >$ for MC at the generated level. This is done in bins of Q, η and ϕ. Specifically,

$$C^{Data/MC}(Q,\eta,\phi) =< 1/p_T^{Data/MC(rec.)}(Q,\eta,\phi) > - < 1/p_T^{MC(gen.)}(Q,\eta,\phi) > \tag{1}$$

$$\frac{1}{p_T^{corrected}} = \frac{1}{p_T} - C^{Data/MC}(Q,\eta,\phi) \;\;\Leftrightarrow\;\; p_i^{corrected} = p_i \times \frac{1.0}{1.0 - p_T * C^{Data/MC}(Q,\eta,\phi)} \tag{2}$$

where, MC(rec.) and MC(gen.) is the information of momentum of the MC at the reconstructed and generated levels, and p_i is the momentum of the muon in x, y, and z direction ($i = x,\ y,\ z$). Here, $C^{Data/MC}$ is the correction factor for the data or MC in bins of Q, η, and ϕ of the muon (8 $\times$ 8 matrix in η and ϕ for each muon polarity). This $< 1/p_T >$ correction corrects for the charge, η, and ϕ dependence of the mis-reconstructed momentum.

Figure 8 shows the $< 1/p_T >$ correction for the data and for the MC ($C^{Data/MC}(Q,\eta,\phi)$) .

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Figure 1: Reference plots BEFORE any corrections. Top Plots: Comparison of the dimuon invariant mass distribution between the data (black) and MC (blue) (left), and the ratio of data to MC (right). Middle plots: Comparison of A_{fb} as a function of mass (left) and boson P_T distribution (right) between the data (black) and MC (blue). Bottom plots: Comparison of ϕ in the Collins-Soper frame for boson $P_T < 5\ GeV/c$ (left), and ϕ in the Collins-Soper frame for boson $5 < P_T < 10\ GeV/c$ (right) for data (black) and MC (blue). The plots are normalized to the total number of events of the data in the $60 < M_{\mu\mu} < 120\ GeV/c^2$ mass range. (a) The top two plots indicate that the location of the Z peak in mass is incorrect and the shape of the data in mass is different from the MC. (b) The left middle plot shows unphysical wiggles in A_{fb} in both data and MC, indicating that the momentum scales for positive and negative muons are different in both data and MC (c) The right middle plot shows that the MC does not have the correct P_T spectrum. (d) The bottom two plots show that the MC does not have the correct P_T spectrum (level) and that the momentum scales for positive and negative muons are different (the peaks at $\phi_{CS} = 0$ and $\pm\pi$ are different).

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Figure 2: Reference plots BEFORE any corrections: Top Plots: Comparison of the average of $M_{\mu\mu}$ as a function of ϕ for μ^- (left) and the average of $M_{\mu\mu}$ as a function ϕ for μ^+ (right) between the data (black) and MC (blue). Bottom Plots: Comparison between the data (black) and MC (blue) of the average of $M_{\mu\mu}$ as a function of η for μ^- (left) and the average of $M_{\mu\mu}$ as a function of η for μ^+ (right). The top two plots show that the average Z mass depends on ϕ (it should be independent of ϕ). They also show that the momentum scales for positive and negative muons are different in both data and MC, and the difference is a function of ϕ of the muon. The bottom two plots show that the η dependence of the muon momentum scales in data and MC are different.

After applying the $< 1/p_T >$ additive correction, we apply global scale factors to the MC to match the Z mass position, and momentum resolutions in data and MC. The three global factors, T, Δ, and SF, are estimated by comparing the overall $M_{\mu^+\mu^-}$ mass distributions between data and MC (using a χ^2 test). These global scale factors are only applied to the MC. They are define by the following equations (which are also used at CDF):

$$p_i^{corrected} = p_i + T \times (p_i^{gen.} - p_i) \tag{3}$$

$$\frac{1}{p_T^{corrected}} = \frac{1}{p_T} + \Delta \times Random :: Gaus(1, SF) \tag{4}$$

where p_i is the reconstructed muon momentum in MC ($i = x$, y, and z) and $p_i^{gen.}$ is the generated muon momentum in MC. Figure 4 shows χ^2 distributions for the comparison of data to MC as a function each global scale factor. The measured global factors (extracted from the χ^2 plot) are summarized in Table 2.

Here T which (is about 0.95) is the factor that scales the resolution in MC to better agree with the data (a change of -5%). Δ is a correction for the overall momentum scale (the P_T of muons fro Z decays is of order 30 GeV, $1/P_T \approx 3 \times 10^{-5}$. Therefore, $\Delta \approx 2 \times 10^{-5}$ is a shift of 0.6%). The parameter SF is an additional resolution smearing in $1/P_T$. Figure 5 and 6 show the reference plots after applying the iteration 1 correction factors,

Table 1: Iteration 1: The global scale factors (T, Δ, and SF) for additional muon momentum correction in the MC. The global factors are determined by comparing the $M_{\mu^+\mu^-}$ distributions in data and MC. These factors are applied only to the MC. Here T is a factor that scales the resolution in MC to better agree with the data. Δ is a correction for the overall momentum scale, and SF is an additional resolution smearing in $1/P_T$.

Global Factor	Value
T	0.9433 ± 0.0020
Δ	$(2.2541 \pm 0.0792) \times 10^{-5}$
SF	10.3708 ± 0.3708

$C(Q, \eta, \phi)$, T, Δ, and SF. The reference plots shows better agreement between the data and MC. The unphysical wiggles in the A_{fb} distributions in both data and MC are no longer there, and the peaks at $\phi_{CS} = 0$ and $\pm\pi$ are of equal magnitude. However, the middle plot shows that Z P_T distribution in MC do not agree with the data. This results in offsets between data and MC in the ϕ_{CS} distributions for the two Z P_T ranges. (The distributions are normalized to the total number of events in data for $60 < M_{\mu^+\mu^-} < 120\ GeV/c^2$ mass window.)

The disagreement [5] between data and MC for the Z P_T distribution at low P_T implies that the Powheg MC generator with Pythia parton showering (used in CMS) should be tuned. In order to get better agreement between the data and MC, we apply a Z P_T correction to the MC at the generator level such that it matches the data.

The Z P_T correction removes the discrepancy in the overall levels in the comparison of ϕ_{CS} distributions between the data and MC for the two low P_T ranges. Figure 7 shows the reference plots after applying both the momentum correction and the additional Z P_T correction in MC. With the additional Z P_T correction, there is agreement in the ϕ_{CS} distributions between data and MC.

Table 2: Iteration 2 (final) : The global scale factors (T, Δ, and SF) for additional muon momentum correction in the MC. The global factors are determined by comparing the $M_{\mu^+\mu^-}$ distributions in data and MC. These factors are applied only to the MC. Here T is a factor that scales the resolution in MC to better agree with the data. Δ is a correction for the overall momentum scale, and SF is an additional resolution smearing in $1/P_T$.

Global Factor	Value
T	0.9433 ± 0.0020
Δ	$(2.2541 \pm 0.0792) \times 10^{-5}$
SF	10.3708 ± 0.3708

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Figure 3: Top Plots: The $< 1/p_T >$ correction for data for μ^- (left) and μ^+ (right) in η and ϕ (for iteration 1) . Bottom Plots: The $< 1/p_T >$ correction for MC for μ^- (left) and μ^+ (right) in η and ϕ (for iteration 1.

5 Muon Momentum Correction- Final Iteration

Now that the Z P_T in the Powheg MC generator has been tuned to match the data, we repeat our analysis, and extract updated muon momentum corrections. This is the second and final iteration.

6 Conclusion

Using the Drell-Yan dimuon sample, we extract corrections to the muon momentum that originate from tracking misalignments. The corrections are obtained by using the average $< 1/p_T >$ of muon in bins of charge, η, and ϕ in conjunction with the dimuon invariant mass distributions. Corrections are extracted for both data and MC.

The $M_{\mu^+\mu^-}$, A_{fb}, ϕ_{CS} distributions are used as reference plots to test the procedure, After the application of the muon momentum correction, the reconstruction bias which is a function of charge, η, and ϕ is removed. All kinematic distributions which are used as reference plots show good agreement between the data and MC. The offline code for the muon momentum corrections is now available.

7 Appendix

7.1 Test of the MuscleFit Correction

In this appendix, we show that the MuscleFit for the 2010 data and MC does a poor when applied to the 2011 data and MC.

The MuscleFit correction was the standard method to correct the muon momentum bias in 2010. However, the MuscleFit correction is not yet available for full 2011A data set.

The MuscleFit has been updated up to first 750 pb^{-1} of the 2011A data set [1]. It is not available for the rest of the 2011A data.

A MuscleFit is not available for the 2011A Monte Carlo . However MuscleFits for both data and MC are available for the 2010 samples.

Figure 4: Top Plots: χ^2 distribution as a function of the global factor, T (left) and Δ (right). Bottom Plots: χ^2 distribution as a function of the global factor, SF.

Therefore, as a test, we apply the MuscleFit for first 750 pb^{-1} of the 2011A data set to the entire 2011A data set, and apply the MuscleFit for the 2010 MC version to the 2011A MC.

To test the performance of the MuscleFit , we use the reference plots which are described in Sec. 3. Figure 9 and 10 show the reference plots of the data and Figure 11 and 12 show the reference plots of MC before (black) and after (blue) applying the MuscleFit corrections. MuscleFit in the data improves ϕ dependence of the muon momentum, but does not change the other kinematic distributions. The MuscleFit in MC shifts the mass distribution by $\sim 0.1\ GeV/c^2$ and overcorrects ϕ dependence of the muon momentum. With the MuscleFit correction we still see unphysical wiggles in A_{fb}, which incidates mis-calibration between positive and negative muons. In addition, he peaks in the ϕ(CS) are not equal in magnitude, which also indicates that there is a mis-calibration between positive and negative muons

The 2010 MC (2010 November version) has the different alignment scenario then the 2011A MC (2011 Spring version). Therefore, the MuscleFit for 2010 November version of MC might not work for 2011 Spring version of the MC. We find that this is indeed the case.

We conclude the MuscleFit from the first third of the 2011A data is applied to the full 2011A data set, it improves the ϕ dependence of the muon momentum, but does not remove the distortions in A_{fb}, or ϕ_{CS} distribution. In addition, it does not account for the η dependence of the momentum correction. This study will be repeated when the updated MuscleFit parameters for full data set becomes available. If using the MuscleFit is desirable, we can repeat the study and determine the Rochester momentum corrections that account for the residual mis-alignments that are not fully corrected for in the updated MuscleFit.

7.2 Test of the Application of $C(Q, \eta, \phi)$ Factor as a Multiplicative Correction

A mis-reconstruction of the muon momentum may be caused by track misalignment of the track or uncertainties in the magnetic field. Misalignment in the tracking adds a fixed curvature shift which is equivalent to a fixed shift in $1/p_T$ which is additive. Therefore, a misalignment results in an additive correction and has different corrections for μ^+ and μ^-. On the other hands, an error in the magnetic field effect is proportional to $1/p_T$ and does not have the any charge dependence. Such an error should be corrected for using multiplicative correction. We already apply an overall scale correction (Δ) to the MC to account for any scale differences between the data and MC.

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Figure 5: The reference plots ($M_{\mu^+\mu^-}$, A_{fb}, Z P_T, and ϕ_{CS}) after the application of the iteration 1 additive muon momentum correction. Top Plots: Comparison of the dimuon invariant mass distribution between the data (black) and MC (blue) (left) and its ratio of data to MC (right). Middle plots: Comparison of A_{fb} (left) and boson P_T (right) distributions between the data (black) and MC (blue). Bottom plots: Comparison of ϕ in the Collins-Soper frame in boson $P_T < 5\ GeV/c$ (left) and ϕ in the Collins-Soper frame in boson $5 < P_T < 10\ GeV/c$ (right) distributions between the data (black) and MC (blue). The plots are normalized to the total number of events of the data in $60 < M_{\mu\mu} < 120\ GeV/c^2$.

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Figure 6: The $M_{\mu^+\mu^-}$ profile plot as a function of η and ϕ of μ^+ and μ^- after the iteration 1 muon additive momentum correction. Top Plots: Comparison of the average of $M_{\mu\mu}$ in ϕ of μ^- (left) and the average of $M_{\mu\mu}$ in ϕ of μ^+ (right) between the data (black) and MC (blue). Bottom Plots: Comparison of the average of $M_{\mu\mu}$ in η of μ^- (left) and the average of $M_{\mu\mu}$ in η of μ^+ (right) between the data (black) and MC (blue).

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Figure 7: The reference plots ($M_{\mu^+\mu^-}$, A_{fb}, Z P_T, and ϕ_{CS}) after muon iteration 1 additive momentum correction and Z P_T correction. Top Plots: Comparison of the dimuon invariant mass distribution between the data (black) and MC (blue) (left) and its ratio of data to MC (right). Middle plots: Comparison of A_{fb} (left) and boson P_T (right) distributions between the data (black) and MC (blue). Bottom plots: Comparison of ϕ in the Collins-Soper frame in boson $P_T < 5\ GeV/c$ (left) and ϕ in the Collins-Soper frame in boson $5 < P_T < 10\ GeV/c$ (right) distributions between the data (black) and MC (blue). The plots are normalized to the total number of events of the data in $60 < M_{\mu\mu} < 120\ GeV/c^2$.

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Figure 8: Top Plots: The $< 1/p_T >$ correction for data for μ^- (left) and μ^+ (right) in η and ϕ (for the second/final iteration) . Bottom Plots: The $< 1/p_T >$ correction for MC for μ^- (left) and μ^+ (right) in η and ϕ (for the second/final iteration.

We find that there is a difference in the correction between positive and negative muons. Such a difference can only originate from track misalignment. Therefore, we estimate the correction factor, $C(Q, \eta, \phi)$, and apply it as an additive correction.

In this appendix we show that using the correction factor as a multiplicative, instead of an additive correction, is incorrect and results in a wrong correction for high p_T. We show this using the MC sample, for which we know the true momentum of the high p_T tracks.

Under the multiplicative assumption, the the correction factor, $C(Q, \eta, \phi)$, is the ratio of $< 1/p_T >^{Data/MC(rec.)}$ to $< 1/p_T >^{MC(gen.)}$ which is defined as $C_M(Q, \eta, \phi)$. The following equations explain how to define and apply the correction factor, $C_M(Q, \eta, \phi)$ under the multiplicative assumption.

$$C_M^{Data/MC}(Q, \eta, \phi) =< 1/p_T^{Data/MC(rec.)}(Q, \eta, \phi) > / < 1/p_T^{MC(gen.)}(Q, \eta, \phi) > \tag{5}$$

$$\frac{1}{p_T^{corrected}} = \frac{1}{p_T} \times C_M^{Data/MC}(Q, \eta, \phi) \Leftrightarrow p_i^{corrected} \ = \ \frac{p_i}{C_M^{Data/MC}(Q, \eta, \phi)} \tag{6}$$

After applying the multiplicative correction, $C_M(Q, \eta, \phi)$, the global factors, T, Δ, and SF are estimated using $M_{\mu^+\mu^-}$ distribution and these global factors are applied into MC. The reference plots for the Z mass region ($60 < M_{\mu^+\mu^-} < 120\ GeV/c^2$) with the multiplicative correction shown in Figure 13 are close to the reference plots with the additive correction.

The multiplicative correction and the additive correction are expected to show a similar effect in the Z boson mass region. However, if the muon has very high momentum e.g. $p_T = 500\ GeV$, then the correction for the two assumptions are very different.

For example, for the case of $C_M(Q, \eta, \phi) = 1 + 0.01$, if applied in multiplicative way, it changes a the momentum of a muon with $p_T^{\mu} = 50\ GeV$ by 1% ($0.5\ GeV$). Similarly, it also changes the momentum of a muon with $p_T^{\mu} = 500\ GeV$ by 1% ($5\ GeV$).

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Figure 9: The effect of the MuscleFit correction (for the first 1/3 of the 2011A data) when applied to the full 2011A data. Top Plots: Comparison of the dimuon invariant mass distribution before (black) and after (blue) applying MuscleFit (left) and its ratio (right). Middle plots: Comparison of A_{fb} (left) and boson P_T (right) distributions before (black) and after (blue) applying MuscleFit correction. Bottom plots: Comparison of ϕ in the Collins-Soper frame in boson $P_T < 5\ GeV/c$ (left) and ϕ in the Collins-Soper frame in boson $5 < P_T < 10\ GeV/c$ (right) distributions before (black) after (blue) applying the MuscleFit correction.

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Figure 10: The effect of the MuscleFit correction (for the first 1/3 of the 2011A data) when applied to the full 2011A data. Top Plots: Comparison of the average of $M_{\mu\mu}$ as a function of ϕ for μ^- (left) and the average of $M_{\mu\mu}$ as function of ϕ for μ^+ (right) before (black) and after (blue) applying the MuscleFit correction. Bottom Plots: Comparison of the average of $M_{\mu\mu}$ as a function of η for μ^- (left) and the average of $M_{\mu\mu}$ as a function of η for μ^+ (right) before (black) and after (blue) applying the MuscleFit correction.

./MC_eta2_4_musclefit_effect-eps-converted-to.pdf

Figure 11: The effect of using the MuscleFit for the 2010 MC on the 2011A MC sample. Top Plots: Comparison of the dimuon invariant mass distribution before (black) and after (blue) applying MuscleFit (left) and its ratio (right). Middle plots: Comparison of A_{fb} (left) and boson P_T (right) distributions before (black) and after (blue) applying the 2010 MC MuscleFit correction. Bottom plots: Comparison of ϕ in the Collins-Soper frame in boson $P_T < 5\ GeV/c$ (left) and ϕ in the Collins-Soper frame in boson $5 < P_T < 10\ GeV/c$ (right) distributions before (black) after (blue) applying the 2010 MC MuscleFit correction.

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Figure 12: The effect of using the MuscleFit for the 2010 MC on the 2011A MC sample. Top Plots: Comparison of the dimuon invariant mass distribution before (black) and after (blue) applying MuscleFit (left) and its ratio (right). Middle plots: Comparison of A_{fb} (left) and boson P_T (right) distributions before (black) and after (blue) applying the 2010 MC MuscleFit correction. Bottom plots: Comparison of ϕ in the Collins-Soper frame in boson $P_T < 5\ GeV/c$ (left) and ϕ in the Collins-Soper frame in boson $5 < P_T < 10\ GeV/c$ (right) distributions before (black) after (blue) applying the 2010 MC MuscleFit correction.

For an additive correction a $C(Q, \eta, \phi) = 1 + 0.0002$, then it changes the momentum of a muon with $p_T^\mu = 50\,GeV$ by 1% ($0.5\,GeV$), but changes the momentum of muon with $p_T^\mu = 500\,GeV$ by 10% ($50\,GeV$).

We find that we need to apply a muon momentum correction to the MC. We extracted the corrections for MC and data in the same way. However, for the MC sample, since we know the generated momentum, we can test the difference at high P_T between applying the correction factor as a multiplicative or additive correction.

In the MC, we compared the p_T of muons between the reconstructed and the generated level in the very high mass region ($M_{\mu^+\mu^-} > 250\ GeV/c^2$). We use the MC information to determine which procedure a reconstructed momentum which is close to the generated momentum.

The comparison of the reconstructed muon momentum to the generated muon momentum shows that the additive correction yields a reconstructed muon momentum which is closer to the generated muon momentum than the multiplicative correction, as shown in Figure 14.

Figure 15 shows the difference of $< 1/p_T(rec.) >$ and $< 1/p_T(gen.) >$ in the Z mass region, $60 < M_{\mu^+\mu^-} < 120\ GeV/c^2$. Even in the Z mass region, the additive correction yields a reconstructed muon momentum which is close to the generated muon momentum.

References

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Figure 13: Test of a multiplicative momentum corrections. Teference plots ($M_{\mu^+\mu^-}$, A_{fb}, Z P_T, and ϕ_{CS}) after the application of a multiplicative muon momentum correction and the Z P_T correction. Top Plots: Comparison of the dimuon invariant mass distribution between the data (black) and MC (blue) (left) and its ratio of data to MC (right). Middle plots: Comparison of A_{fb} (left) and boson P_T (right) distributions between the data (black) and MC (blue). Bottom plots: Comparison of ϕ in the Collins-Soper frame in boson $P_T < 5\ GeV/c$ (left) and ϕ in the Collins-Soper frame in boson $5 < P_T < 10\ GeV/c$ (right) distributions between the data (black) and MC (blue). The plots are normalized to the total number of events of the data in $60 < M_{\mu\mu} < 120\ GeV/c^2$.

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Figure 14: The difference of $< 1/p_T(rec.) >$ and $< 1/p_T(gen.) >$ in $M_{\mu^+\mu^-} > 250\ GeV/c^2$. Top plot: The difference of $< 1/p_T(rec.) >$ and $< 1/p_T(gen.) >$ after applying the additive correction for μ^- (left) and μ^+ (right). Bottom plot: The difference of $< 1/p_T(rec.) >$ and $< 1/p_T(gen.) >$ after applying the multiplicative correction for μ^- (left) and μ^+ (right).

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Figure 15: The difference of $< 1/p_T(rec.) >$ and $< 1/p_T(gen.) >$ in $60 < M_{\mu^+\mu^-} < 120\ GeV/c^2$. Top plot: The difference of $< 1/p_T(rec.) >$ and $< 1/p_T(gen.) >$ after applying the additive correction for μ^- (left) and μ^+ (right). Bottom plot: The difference of $< 1/p_T(rec.) >$ and $< 1/p_T(gen.) >$ after applying the multiplicative correction for μ^- (left) and μ^+ (right).